Supplement to “Choosing Your Pond: A Structural Model of Political Power Sharing”

Selcen Çakır

August 7, 2020

Appendix A The share equation

This section derives the closed-form solution of the rent share a politician earns in a party by adjusting the steps taken in CPR for the possibility of a U-shaped returns to party size. Substituting a leader's stationary decision rule (equation 3.1) into the value function of a type-z politician with share ϕ in a type-x party (equation 3.7), we get

$$
\begin{aligned}
&[\rho+\delta+\lambda\bar{F}(q_b(\cdot))+\lambda F(q_a(\cdot))]V(z,\phi,\phi^{l^*}(z,x),x) \\
&=\phi\frac{z}{x}\theta(x)+\psi(x)+\delta V_0(z) \\
&+\lambda[F(x_a(\cdot))+\bar{F}(x_b(\cdot))]V(z,\phi^{l^*}(z,x),\phi^{l^*}(z,x),x)+\lambda\int_{q_b(\cdot)}^{x_b(\cdot)}V(z,\phi^{l^*}(z,m),\phi^{l^*}(z,m),m)dF(m) \\
&+\lambda\int_{x_a(\cdot)}^{q_a(\cdot)}V(z,\phi^{l^*}(z,m),\phi^{l^*}(z,m),m)dF(m). \qquad \text{(A.1)}
\end{aligned}
$$

To obtain the share equation, first, we use integration by parts in equation A.1 to get an expression for $V(z,\phi,\phi^{l^*}(z,x),x)$. Next, we get another representation for $V(z,\phi,\phi^{l^*}(z,x),x)$ by using the leader's stationary decision rules (equations 3.1). Equating these two expressions allows for obtaining the closed-form solution of the share equation.

Using integration by parts in equation A.1 and simplifying terms, we obtain

$$
\begin{aligned}
(\rho+\delta)V(z,\phi,\phi^{l^*}(z,x),x) &= \phi\frac{z}{x}\theta(x)+\psi(x)+\delta V_0(z) \\
&+ \lambda\int_{q_b(\cdot)}^{x_b(\cdot)}\frac{dV(z,\phi^{l^*}(z,m),\phi^{l^*}(z,m),m)}{dm}\bar{F}(m)dm \\
&- \lambda\int_{x_a(\cdot)}^{q_a(\cdot)}\frac{dV(z,\phi^{l^*}(z,dm),\phi^{l^*}(z,m),dm)}{dm}F(m)dm. \qquad \text{(A.2)}
\end{aligned}
$$

Now, suppose that the politician's outside option is a type-x' party. Following the leader's stationary decision rule in equation 3.1, we have

$$(\rho+\delta)V(z,\phi^l(z,x,x',\phi^{l^*}(z,x),\phi^{l^*}(z,x')),x) = (\rho+\delta)V(z,\phi^{l^*}(z,x'),\phi^{l^*}(z,x'),x'),$$

which, using equations 3.1 and 3.8, can be rewritten as

$$(\rho+\delta)V(z,\phi^l(z,x,x',\phi^{l^*}(z,x),\phi^{l^*}(z,x')),x) = \phi^{l^*}(z,x')\frac{z}{x'}\theta(x') + \psi(x') + \delta V_0(z). \quad \text{(A.3)}$$

Equating the right-hand-side of equation A.2 with that of equation A.3 gives the equilibrium share $\phi^l(z,x,x',\phi^{l^*}(z,x),\phi^{l^*}(z,x'))$ that convinces the politician to join a type-x party when his outside option is membership in a type-x' party (equation 3.11).

Appendix B Steady-state flow equalities

This section derives the steady-state flow equalities by adjusting the steps taken in CPR for the possibility of a U-shaped returns to party size.

- *The proportion of independent politicians*

  Let φ_z denote the proportion of type-z independent politicians. The flows into the stocks of independent type-z politicians are due to exogenous match break-ups, which occur at rate $M\ell(z)(1-\varphi_z)\delta$. The outflows from the stocks of independent type-z politicians occur as they get an acceptable offer, which occurs at rate $M\ell(z)\varphi_z\lambda[F(x_{a0}(\cdot)) + \bar{F}(x_{b0}(\cdot))]$. In a steady-state, the flows into and outflows from the stocks of independent politicians are equal, which gives the proportion of type-z independent politicians,

$$\varphi_z = \frac{\delta}{\delta + \lambda[F(x_{a0}(\cdot)) + \bar{F}(x_{b0}(\cdot))]}. \quad \text{(B.1)}$$

- *The joint density of type-z politicians in type-x parties*

  Consider a medium type-z politician. Suppose that $x > x_0(z)$, i.e., the politician considers a type-x party as a big party. The outflows from the stocks of politicians of type-z, member of parties of type-x, and paid less than $\phi \in [\phi(z,x,0,\phi^{l^*}(z,x),0),\phi^{l^*}(x)]$, denoted $\Gamma_{\phi|z,x}(\phi|z,x)g(z,x|\Phi^{l^*}(z,x))M(1-\varphi_z)$, leave this category in either of two ways. First, the match exogenously breaks up at rate δ. Second, they receive an offer from a party of type $x' \in [x_{min}, q_a(\cdot)] \cup [q_b(\cdot), x^{max}]$ that either causes a share improvement or induces them to leave their current party, which occurs at rate $\lambda[F(q_a(\cdot)+\bar{F}(q_b(\cdot)]$. The politicians enter this category either by switching from parties of type-$x' \in [q_a(\cdot), q_b(\cdot)]$ or from the pool of independents. The steady-state equality between flows into and outflows from the stocks $\Gamma_{\phi|z,x}(\phi|z,x)g(z,x|\Phi^{l^*}(z,x))M(1-\varphi_z)$ is

$$\begin{aligned}[\delta + \lambda[\bar{F}(q_b(\cdot)) + F(q_a(\cdot))]]M(1-\varphi_z)\Gamma_{\phi|z,x}(\phi|z,x)g(z,x|\Phi^{l^*}(z,x)) \\ = \lambda M\varphi_z \ell(z)f(x) + \lambda f(x)M(1-\varphi_z)\int_{q_a(\cdot)}^{q_b(\cdot)} g(z,m|\Phi^{l^*}(z,x))dm \end{aligned} \tag{B.2}$$

Evaluating equation B.2 at $\phi = \phi^{l^*}(z,x)$, (which has the property that $\Gamma_{\phi|z,x}(\phi^{l^*}(z,x)|z,x) = 1$, $q_b(\cdot) = x$, and $q_a(\cdot) = x_a(\cdot)$), and using straightforward algebra, we obtain the joint density of type-z politicians in parties of types x and $x_a(\cdot)$ as

$$g(z,x|\Phi^{l^*}(z,x)) = \frac{\delta(\delta+\lambda)}{\left[\delta + \lambda[\bar{F}(x) + F(x_a(\cdot))\right]^2}\tilde{\ell}(z)f(x) \tag{B.3}$$

and

$$g(z,x_a(\cdot)|\Phi^{l^*}(z,x)) = \frac{\delta(\delta+\lambda)}{\left[\delta + \lambda[\bar{F}(x) + F(x_a(\cdot))\right]^2}\tilde{\ell}(z)f(x_a(\cdot)) \tag{B.4}$$

respectively, where

$$\tilde{\ell}(z) = \frac{\ell(z)}{F(x_{a0}(\cdot)) + \bar{F}(x_{b0}(\cdot))} \tag{B.5}$$

is defined as the *effective density* of type-z politicians, as it weights the politician's density by its demand by the parties. Note that the joint density of a politician in a party decrease in both the politician's probability of getting an acceptable offer conditional on getting an offer, $F(x_{a0}(\cdot)) + \bar{F}(x_{b0}(\cdot))$, and the probability of getting an offer from a party that the politician ranks better than types-x and $x_a(\cdot)$ parties, $\lambda[\bar{F}(x) + F(x_a(\cdot))]$, due to increased competition by the parties.

- *The joint density of type-$(z, q_b(\cdot))$ politicians and type-x parties*

Consider a medium type-(z,x') politician in a type-x party. Suppose that both x and x' are "big" parties for the politician. Note that the politician's thresholds for switching to another party and having a share improvement in the party are $x_a(\cdot)$,$x_b(\cdot)$ and $q_a(\cdot)$,$q_b(\cdot)$, respectively. Moreover, when member of a big party, he ranks all bigger parties better, thus $x_b(\cdot) = x$. Similarly, when his outside option is a big party, an offer from a party that is bigger than his outside option and smaller than his bigger-party switching threshold cause a share improvement in the party, and, hence, $q_b(\cdot) = x'$. The outflows from the stocks of type-$(z, q_b(\cdot))$ politicians, member of parties of type-x, and paid $\phi^l(z, x, q_b(\cdot), \phi^{l^*}(z, x), \phi^{l^*}(z, q_b(\cdot)))$ leave this category in either of two ways. First, the match exogenously breaks up at rate δ. Second, they get an offer from a party of type $x'' \in \{[x_{min}, q_a(\cdot)] \cup [q_b(\cdot), x^{max}]\}$ that either causes a share improvement or induces them to leave their party, which occurs at rate $\lambda[F(q_a(\cdot)) + \bar{F}(q_b(\cdot)]$. The politicians enter this category in either of two ways. First, they switch from parties of type-$x'' \in \{q_a(\cdot), q_b(\cdot)\}$. Second, if they were already a member of a type-x party and had a worse outside option than $q_b(\cdot)$, they get an offer from an outside party of type-$x' \in \{q_a(\cdot), q_b(\cdot)\}$. Then, the steady-state equality between flows into and outflows from the stocks $M(1-\varphi_z)\mu_{z,q_b(\cdot),x}(z, q_b(\cdot), x|\Phi^{l^*}(z, x))$ is

$$\begin{aligned}
&[\delta + \lambda[\bar{F}(q_b(\cdot)) + F(q_a(\cdot)]]M(1-\varphi_z)\mu_{z,q_b(\cdot),x}(z, q_b(\cdot), x|\Phi^{l^*}(z, x)) \\
&\quad = \lambda M(1-\varphi_z)f(x)g(z, q_b(\cdot)|\Phi^{l^*}(z, x)) \\
&\quad + \lambda M(1-\varphi_z)f(q_b(\cdot))\int_{q_a(\cdot)}^{q_b(\cdot)} \mu_{z,m,x}(z, m, x|\Phi^{l^*}(z, x))dm
\end{aligned} \tag{B.6}$$

which, after using straightforward algebra and rearranging the terms, yields the joint density of type-$(z, q_b(\cdot))$ and type-$(z, q_a(\cdot))$ politicians and type-x parties,

$$\mu_{z,q_b(\cdot),x}(z, q_b(\cdot), x|\Phi^{l^*}(z, x)) = 2\frac{\delta(\delta+\lambda)\lambda f(x)\tilde{\ell}(z)f(q_b(\cdot))}{[\delta + \lambda[\bar{F}(q_b(\cdot)) + F(q_a(\cdot))]]^3}, \tag{B.7}$$

and

$$\mu_{z,q_a(\cdot),x}(z, q_a(\cdot), x|\Phi^{l^*}(z, x)) = -2\frac{\delta(\delta+\lambda)\lambda f(x)\tilde{\ell}(z)f(q_a(\cdot))}{\Big[[\delta + \lambda[\bar{F}(q_b(\cdot)) + F(q_a(\cdot))]]\Big]^3}. \tag{B.8}$$

respectively.

- *The joint density of type-$(z, 0)$ politicians and type-x parties*

The flows into this category occurs as a type-x leader meets a type-z independent politician at rate λ. The outflows occur either through an exogenous match break up, occurring at rate δ, or when a politician gets an offer from a party of type $x' \in \{[x_{min}, x_{a0}(\cdot) \cup [x_{b0}(\cdot), x^{max}]\}$ that either induces the politician to switch the party or improves his outside option in the party. The steady-state equality of the flows into and the outflows from the stocks $M(1-\varphi)\mu_{z,0,x}(z,0,x|\Phi^{l^*}(z,x))$ is

$$\varphi M\lambda\ell(z)f(x) = M(1-\varphi)\mu_{z,0,x}(z,0,x|\Phi^{l^*}(z,x))[\delta+\lambda[F(x_{a0}(z)+\bar{F}(x_{b0}(z))]]$$

which, after simplifying the M term and imposing $\lambda\varphi_z = \frac{\delta(1-\varphi_z)}{[F(x_{a0}(z))+\bar{F}(x_{b0}(z))]}$ becomes

$$\mu_{z,0,x}(z,0,x|\Phi^{l^*}(z,x)) = \frac{\delta}{[\delta+\lambda[F(x_{a0}(z)+\bar{F}(x_{b0}(z))]]}\tilde{\ell}(z)f(x) \tag{B.9}$$

Appendix C The unconditional likelihood function

This section derives the unconditional likelihood of observing a party affiliation duration following Ridder and van den Berg (2003). Let d_n, d_r, and d_i denote the indicator functions for the uncensored, right-censored, and interval-censored observations, respectively. I begin by deriving the likelihood contribution of the uncensored observations, and then present the contributions of the censored observations. Since the low, medium, and high politician types follow different decision rules for switching a party, the likelihood function takes the probability of the politician belonging to a particular type into account. Formally, the unconditional likelihood of a membership duration of t for an uncensored observation is

$$\begin{aligned} p(t|d_n=1) &= L(\underline{z})p(t|z\le\underline{z},d_n=1)+(L(\bar{z})-L(\underline{z}))p(t|z\in\{\underline{z},\bar{z}\},d_n=1) \\ &+ (1-L(\bar{z}))p(t|z\ge\bar{z},d_n=1), \end{aligned} \tag{C.1}$$

where $\underline{z}$ and $\bar{z}$ are the threshold politician types that separate the low and the high types from the medium types of politicians, respectively.

Since all party transition processes are Poisson, all corresponding durations are exponentially distributed. The rate at which a low-type politician leaves a type-x party is $\delta[1+\kappa\bar{F}(x)]$. Thus, the density of a membership spell of t in a type-x party for a low type-z politician is

$$p(t|z\le\underline{z},x,d_n=1) = \delta[1+\kappa\bar{F}(x)]e^{-\delta[1+\kappa\bar{F}(x)]t}. \tag{C.2}$$

I treat the party type as unobserved heterogeneity and integrate equation C.2 over the density of the party types, $g(x|z,\Phi^{l^*}(z,x)) = \frac{1+\kappa}{[1+\kappa\bar{F}(x)]^2}f(x)$, which was derived in equation

3.13. So, the likelihood of observing a party affiliation spell of t for a low-type politician is

$$
\begin{aligned}
p(t|z \leq \underline{z}, d_n = 1) &= p(t|z \leq \underline{z}, x, d_n = 1) g(x|z, \Phi^{l^*}(z,x)) \\
&= \int_{x_{min}}^{x^{max}} \delta[1+\kappa \bar{F}(x)] e^{-\delta[1+\kappa\bar{F}(x)]t} \frac{1+\kappa}{[1+\kappa[\bar{F}(x)]^2} f(x) dx \\
&= \frac{\delta(1+\kappa)}{\kappa} \int_1^{1+\kappa} \frac{e^{-\delta a t}}{a} da, \qquad \text{(C.3)}
\end{aligned}
$$

where $a = 1+\kappa\bar{F}(x)$ is the probability of leaving a type-x party as a fraction of the probability of having a need for new party membership, δ.

The hazard of leaving a type-x party for a high type-z politician is $\delta[1+\kappa F(x)]$, and the joint density of high type-z politicians in type-x parties is $g(z, x|\Phi^{l^*}(z,x)) = \frac{1+\kappa}{[1+\kappa\bar{F}(x)]^2}\tilde{\ell}(z) f(x)$ (equation 3.13). So, the likelihood of observing a membership spell of t for him is

$$
\begin{aligned}
p(t|z \geq \bar{z}, d_n = 1) &= p(t|z \geq \bar{z}, x, d_n = 1) g(x|z, \Phi^{l^*}(z,x)) \\
&= \int_{x_{min}}^{x^{max}} \delta[1+\kappa F(x)] e^{-\delta[1+\kappa F(x)]t} \frac{1+\kappa}{[1+\kappa F(x)]^2} f(x) dx \\
&= \frac{\delta(1+\kappa)}{\kappa} \int_1^{1+\kappa} \frac{e^{-\delta a t}}{a} da, \qquad \text{(C.4)}
\end{aligned}
$$

where $a = 1+\kappa F(x)$.

Recall that a medium type-z politician has a threshold party type $x_0(z)$ such that he considers all smaller parties than $x_0(z)$ as small, and the others as big. Due to the U-shaped returns to party size, he may consider two parties with different sizes of equal value. Accordingly, when a type-z politician is member of a small type-x party, he is better-off in all smaller parties than the current party and all parties that are larger than his bigger party-switching threshold, $x_b(z, x)$. Then, the hazard of leaving a small type-x party is $\delta[1+\kappa[\bar{F}(x_b(z,x)) + F(x)]]$, and the joint density of medium type-z politicians in type-x parties is $g(z, x|\Phi^{l^*}(z,x)) = \frac{1+\kappa}{[1+\kappa[\bar{F}(x_b(z,x))+F(x)]]^2}\tilde{\ell}(z) f(x)$ (equation 3.17). Similarly, when $x > x_0(z)$, the hazard of leaving a type-x party is $\delta[1+\kappa\bar{F}(x) + \kappa F(x_a(z,x))]$, and the joint density of medium type-z politicians in type-x parties is $g(z, x|\Phi^{l^*}(z,x)) = \frac{1+\kappa}{[1+\kappa\bar{F}(x_b(z,x))+\kappa F(x)]^2}\tilde{\ell}(z) f(x)$.

Thus, the likelihood of observing a party affiliation spell of t for a medium-type politician is

$$\begin{aligned}
p(t|z \in \{\underline{z}, \bar{z}\}, d_n = 1) &= p(t|z \in \{\underline{z}, \bar{z}\}, x, d_n = 1) g(x|z, \Phi^{l^*}(z, x)) \\
&= \int_{x_{min}}^{x_0(z)} \delta[1 + \kappa[\bar{F}(x_b(z, x)) + F(x)]] e^{-\delta[1+\kappa[\bar{F}(x_b(z,x))+F(x)]]t} \\
&\times \frac{1 + \kappa}{[1 + \kappa[\bar{F}(x_b(z, x)) + F(x)]]^2} f(x) dx \\
&- \int_{x^{max}}^{x_0(z)} \delta[1 + \kappa[\bar{F}(x) + F(x_a(z, x))]] e^{-\delta[1+\kappa[\bar{F}(x)+F(x_a(z,x))]]t} \\
&\times \frac{1 + \kappa}{[1 + \kappa[\bar{F}(x) + F(x_a(z, x))]]^2} f(x) dx. \qquad \text{(C.5)}
\end{aligned}$$

Now, suppose that $x_b(z, x_{min}) < x^{max}$, i.e., no smaller party provides a greater value to the politician when he is a member of a type-$x_b(z, x_{min})$ party. Accordingly, the politician behaves like a low-type over the range $[x_b(z, x_{min}), x^{max}]$. Note that

$$\begin{aligned}
&\int_{x_{min}}^{x_0(z)} \delta[1 + \kappa[\bar{F}(x_b(z, x)) + F(x)]] e^{-\delta[1+\kappa[\bar{F}(x_b(z,x))+F(x)]]t} \times \frac{1 + \kappa}{[1 + \kappa[\bar{F}(x_b(z, x)) + F(x)]]^2} f(x) dx \\
&- \int_{x_b(z,x_{min})}^{x_0(z)} \delta[1 + \kappa[\bar{F}(x) + F(x_a(z, x))]] e^{-\delta[1+\kappa[\bar{F}(x)+F(x_a(z,x))]]t} \times \frac{1 + \kappa}{[1 + \kappa[\bar{F}(x) + F(x_a(z, x))]]^2} f(x) dx \\
&= \int_{x_{min}}^{x_0(z)} \delta[1 + \kappa[\bar{F}(x_b(z, x)) + F(x)]] e^{-\delta[1+\kappa[\bar{F}(x_b(z,x))+F(x)]]t} \\
&\times \frac{1 + \kappa}{[1 + \kappa[\bar{F}(x_b(z, x)) + F(x)]]^2} [f(x) - f(x_b(z, x))] dx. \qquad \text{(C.6)}
\end{aligned}$$

Substituting equation C.6 into equation C.5, one obtains

$$\begin{aligned}
p(t|z \in \{\underline{z}, \bar{z}\}, d_n = 1) = &\int_{x_{min}}^{x_0(z)} \delta[1 + \kappa[\bar{F}(x_b(z, x)) + F(x)]] e^{-\delta[1+\kappa[\bar{F}(x_b(z,x))+F(x)]]t} \\
&\times \frac{1 + \kappa}{[1 + \kappa[\bar{F}(x_b(z, x)) + F(x)]]^2} [f(x) - f(x_b(z, x))] dx \\
&- \int_{x^{max}}^{x_b(z,x_{min})} \delta[1 + \kappa\bar{F}(x)] e^{-\delta[1+\kappa\bar{F}(x)]t} \times \frac{1 + \kappa}{[1 + \kappa\bar{F}(x)]^2} f(x) dx. \qquad \text{(C.7)}
\end{aligned}$$

Applying change of variable in the first term with $a = 1 + \kappa[\bar{F}(x_b(z, x)) + F(x)]$, $da = \kappa[f(x) - f(x_b(z, x)) \frac{dx_b(z,x)}{dx}] dx = \kappa[f(x)dx - f(x_b(z, x)) dx_b(z, x)]$, and in the second term with

$a = 1 + \kappa \bar{F}(x), da = -\kappa f(x)dx$, one gets

$$
\begin{aligned}
p(t|z \in \{\underline{z}, \bar{z}\}, d_n = 1) &= \frac{\delta(1+\kappa)}{\kappa} \int_{1+\kappa\bar{F}(x_b(z,x_{min}))}^{1+\kappa} \frac{e^{-\delta at}}{a} da + \frac{\delta(1+\kappa)}{\kappa} \int_{1}^{1+\kappa\bar{F}(x_b(z,x_{min}))} \frac{e^{-\delta at}}{a} da \\
&= \frac{\delta(1+\kappa)}{\kappa} \int_{1}^{1+\kappa} \frac{e^{-\delta at}}{a} da. \qquad \text{(C.8)}
\end{aligned}
$$

Finally, substituting equations C.3, C.4, and C.8 into equation C.1, the unconditional likelihood of a membership duration of t for an uncensored observation is

$$
\begin{aligned}
p(t|d_n = 1) &= L(\underline{z})p(t|z \leq \underline{z}, d_n = 1) + (L(\bar{z}) - L(\underline{z}))p(t|z \in \{\underline{z}, \bar{z}, d_n = 1\}) \\
&+ (1 - L(\bar{z})p(t|z \geq \bar{z}, d_n = 1) \\
&= \frac{\delta(1+\kappa)}{\kappa} \int_{1}^{1+\kappa} \frac{e^{-\delta at}}{a} da. \qquad \text{(C.9)}
\end{aligned}
$$

There are three sources of right-censorship in data: death, the Constitutional Court banning the politician from affiliating with a political party (which is the case for only a few observations), and the politician being a member of a party in the last period of data. The likelihood contribution of a right-censored observation is the probability that the membership did not end until the censoring time. Adjusting the unconditional likelihood function for right-censoring is straightforward and derivation is skipped from this appendix for brevity. The unconditional likelihood of membership duration of t for a right-censored observation is

$$
\begin{aligned}
p(t|d_r = 1) &= L(\underline{z})p(t|z \leq \underline{z}, d_r = 1) + (L(\bar{z}) - L(\underline{z}))p(t|z \in \{\underline{z}, \bar{z}, d_r = 1\}) \\
&+ (1 - L(\bar{z})p(t|z \geq \bar{z}, d_r = 1) \\
&= \frac{(1+\kappa)}{\kappa} \int_{1}^{1+\kappa} \frac{e^{-\delta at}}{a^2} da. \qquad \text{(C.10)}
\end{aligned}
$$

Interval censoring occurs when a member of a parliament loses an election, but reappears on the ballot lists of a different party in a consecutive election. The likelihood contribution of an interval-censored observation is the probability that the membership ended over the interval $T \in (t_1, t_2)$. Adjusting the unconditional likelihood function for interval-censoring is straightforward and derivation is skipped from this appendix for brevity. The likelihood contribution of an interval-censored observation is

$$\begin{aligned} p(t|d_i = 1) &= L(\underline{z})p(t|z \leq \underline{z}, d_i = 1) + (L(\bar{z}) - L(\underline{z}))p(t|z \in \{\underline{z}, \bar{z}, d_i = 1\}) \\ &+ (1 - L(\bar{z})p(t|z \geq \bar{z}, d_i = 1) \\ &= \frac{(1+\kappa)}{\kappa} \int_1^{1+\kappa} \frac{e^{-\delta a t_2} \_ {}^{-\delta a t_1}}{a^2} da \end{aligned} \tag{C.11}$$

where the second equality substitutes equations C.14-C.16. Accordingly, the unconditional likelihood of observing a membership duration of t is

$$\begin{aligned} p(t) &= p(t|d_n = 1)^{d_n} \times p(t|d_r = 1)^{d_r} \times p(t|d_i = 1)^{d_1} \\ &= \Big(\frac{\delta(1+\kappa)}{\kappa} \int_1^{1+\kappa} \frac{e^{-\delta a t}}{a} da\Big)^{d_n} \Big(\frac{1+\kappa}{\kappa} \int_1^{1+\kappa} \frac{e^{-\delta a t}}{a^2} da\Big)^{d_r} \Big(\frac{1+\kappa}{\kappa} \int_1^{1+\kappa} \frac{e^{-\delta a t_2} - e^{-\delta a t_1}}{a^2} da\Big)^{d_i} \end{aligned} \tag{C.12}$$

where the second equality subsitutes equations C.9, C.10, and C.11.

Appendix D Distribution of Politicians' Occupations Across Parties

This section provides details on the distribution of politicians' occupations across parties. In Figure 1, I divide parties into three groups by their estimated sizes (see Figure 2 in the main text). I consider the 4 parties with the largest estimated sizes as big, the next 6 largest parties as medium, and 23 smallest parties as small. The figure shows that the largest 4 parties are home to a large fraction of the politicians with good labor market outcomes. For example, about 47% of the bureaucracts, 37% of all politicians with legal occupations, and 39% of the healthcare practitioners are members of the largest parties. Recall that my data contains politicians who appeared in party ballot lists. Because all parties have equal number of positions in the ballot lists, Figure 1 indicates that, on average, politicians on the ballot lists of the large parties have better labor market options.

Figure 2 compares politicians' occupations across left-wing and right-wing parties as well as the outlier right-wing party whose size is four times as big as the next biggest party. The figure shows that the outlier party is home to a sizeable proportion of the politicians with good labor market options. For example, about 7% of all healthcare practitioners, 9% of politicians with legal occupations, and 8% of politicians with occupations in life, physical, and social sciences are members of the largest party.

Figure 1: The Distribution of Politicians' Occupations Across Parties of Different Sizes

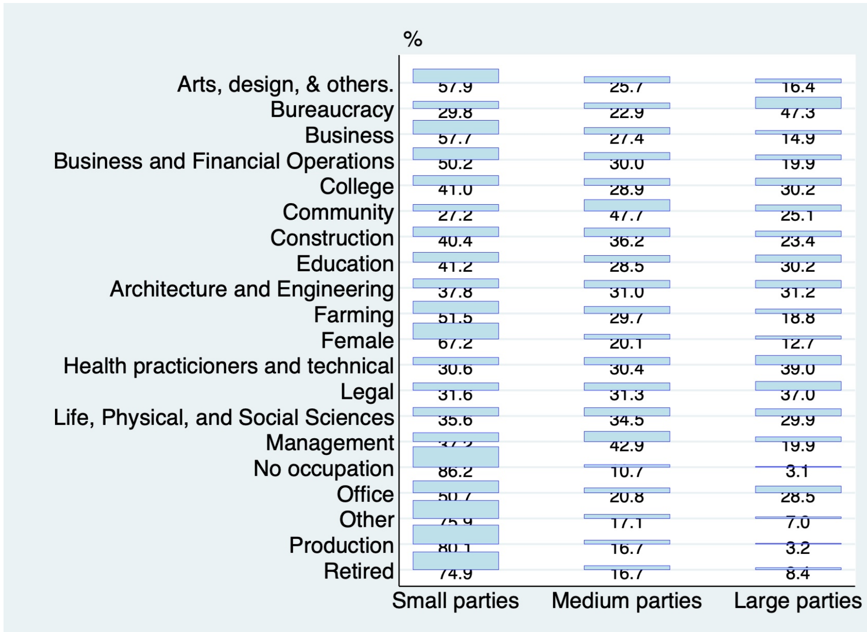
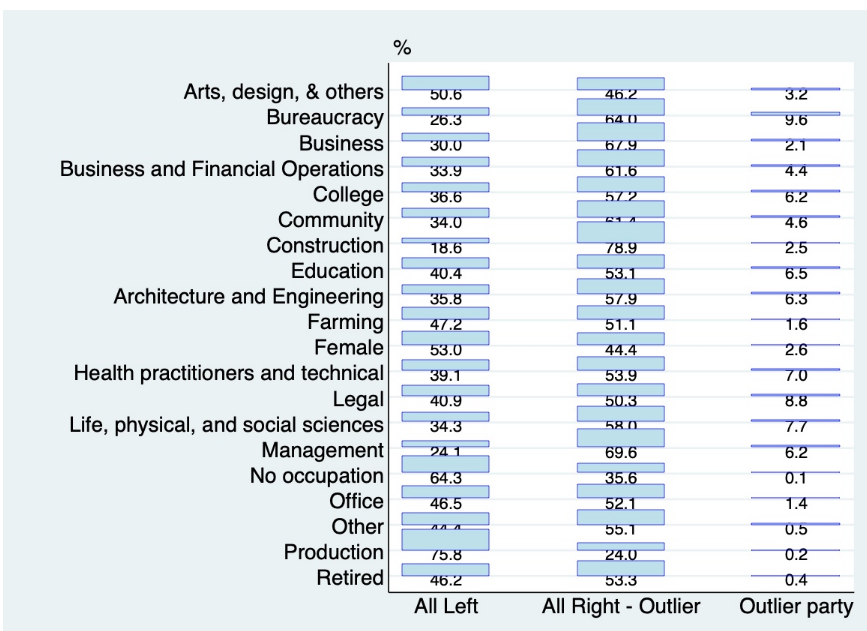


Figure 2: The Distribution of Politicians' Occupations Across Parties of Different Ideologies



Appendix E Comparative Statics

This section shows three results.

1. The rent share that convinces a politician to join a party decreases in party's size.
2. A more resourceful politician demands more rents for joining a party.
3. A party leader's profits (total rents) increases in her party's size.

The first result shows how a liberal democracy can be vulnerable to strong party leaders, as bigger parties' leaders can more easily control their parties. The second result implies that a rent-seeking party leader aims to fill her party with poor-quality politicians who demand little rents to join the party. The third result shows that the distribution of party sizes is equal to the exogenous distribution of leadership abilities.

The first two results are shown by making the following assumption.

Assumption 1. $(1+\rho)\theta(z^{max}) - \psi(x_{min}) < 0$

This assumption can be summarized as follows: even the smallest party has more resources than its most-resourceful member. Note that this assumption states only a sufficient and not a necessary condition.

E.1 A politician's rent share

E.1.1 $\frac{d\phi^l(z,x,0,\phi^{l^*}(z,x),0)}{dx}$

The rent share a type-x party leader pays to a type-$(z,0)$ politician is

$$\begin{aligned}\phi^l(z,x,0,1,0) &= [\rho V_0(z) - \psi(x)]\frac{x}{z\theta(x)} - \frac{x}{z\theta(x)}\lambda\Big\{\int_{x_{b0}(\cdot)}^{x} \underbrace{\frac{dV(z,1,1,m)}{dm}}_{>0} \bar{F}(m)dm \\ &- \int_{x_a(\cdot)}^{x_{a0}(\cdot)} \underbrace{\frac{dV(z,1,1,m)}{dm}}_{<0} F(m)dm\Big\} \qquad \text{(E.1)}\end{aligned}$$

with derivative

$$
\begin{aligned}
\frac{d\phi^l(z,x,0,\phi^{l^*}(z,x),0)}{dx} = \quad & \underbrace{\frac{d}{dx}\Big(\frac{x}{z\theta(x)}\Big)}_{>0}\Big\{\rho V_0(z) - \psi(x) - \lambda\int_{x_{b0}(\cdot)}^{x}\underbrace{\frac{dV(z,1,1,m)}{dm}}_{>0}\bar{F}(m)dm \\
+ \ & \lambda\int_{x_a(\cdot)}^{x_{a0}(\cdot)}\underbrace{\frac{dV(z,1,1,m)}{dm}}_{<0}F(m)dm\Big\} \\
+ \ & \frac{x}{z\theta(x)}\Big\{-\underbrace{\psi'(x)}_{>0} - \underbrace{\frac{dx_b(\cdot)}{dx}}_{\substack{=1 \text{ if } x \text{ is a big party} \\ <0 \text{ if } x \text{ is a small party}}}\lambda\underbrace{\frac{dV(z,1,1,x_b(\cdot))}{dx_b(\cdot)}}_{>0}\bar{F}(x_b(\cdot)) \\
- \ & \underbrace{\frac{dx_a(\cdot)}{dx}}_{\substack{<0 \text{ if } x \text{ is a big party} \\ =1 \text{ if } x \text{ is a small party}}}\lambda\underbrace{\frac{dV(z,1,1,x_a(\cdot))}{dx_a(\cdot)}}_{<0}F(x_a(\cdot))\Big\} \qquad \text{(E.2)}
\end{aligned}
$$

A sufficient condition for the first three lines in equation E.2 to be negative is $\rho V_0(z) = (1+\rho)\theta(z) < \psi(x)$. If a type-$z$ politician considers a type-x party as a big party, then $x_b(\cdot) = x$ and $\frac{dx_b(\cdot)}{dx} = 1$ and $\frac{dx_a(\cdot)}{dx} < 0$. As a result, the last two lines of equation E.2 are also negative. This implies that, among the parties that a type-z politician considers as big, the bigger parties can extract more rents from him. This result is important for understanding how liberal democracies can be vulnerable to strong party leaders.[1]

Before showing the next comparative static, note that equation E.2 can be rewritten as

$$
\begin{aligned}
\frac{d\phi^l(\cdot)}{dx} \ = \ & \frac{d}{dx}\Big(\frac{x}{z\theta(x)}\Big)\frac{z\theta(x)}{x}\phi^l(z,x,0,\phi^{l^*}(z,x),0) + \frac{x}{z\theta(x)}\Big\{-\psi'(x) \\
- \ & \frac{dx_b(\cdot)}{dx}\lambda\frac{dV(z,1,1,x_b(\cdot))}{dx_b(\cdot)}\bar{F}(x_b(\cdot))\frac{dx_a(\cdot)}{dx}\lambda\frac{dV(z,1,1,x_a(\cdot))}{dx_a(\cdot)}F(x_a(\cdot))\Big\}, \quad \text{(E.3)}
\end{aligned}
$$

which I will in section A.2.

E.1.2 $\frac{d\phi^l(z,x,0,\phi^{l^*}(z,x),0)}{dz}$

In this section, I show that the rent share a type-x party leader pays to a type-$(z,0)$ politician (equation E.1) is increasing in politician's assets. I do this separately for parties that are considered as "big" and "small" by a type-z politician.

[1] If a type-z politician considers a type-x party as a small party, then $x_a(\cdot) = x_a$ and $\frac{dx_b(\cdot)}{dx} < 0$. So, the last two terms of equation E.2 in this case are positive. This implies that, among the parties that a typr-z politician considers as small, the smaller parties *may* pay less rents to the politician, depending on which of the terms in equation E.2 dominate.

Case 1: Type-x party is considered big by type-z politician

Substituting $V_0(z) = \frac{1+\rho}{\rho}$ into E.1 and taking the derivative with respect to z, we obtain

$$
\begin{aligned}
\frac{\partial \phi^l(\cdot)}{\partial z} &= \underbrace{-\frac{x}{z^2\theta(x)}[(1+\rho)\theta(z)-\psi(x)]}_{>0 \text{ by assumption 1}} + \underbrace{\frac{x}{z\theta(x)}(1+\rho)\theta'(z)}_{>0} \\
&+ \frac{x}{z^2\theta(x)}\lambda\Big(\int_{q_b(\cdot)}^{x_b(\cdot)} \underbrace{\frac{dV(\psi_{z,x,m})}{dm}}_{>0}\bar{F}(m)dm - \int_{x_a(\cdot)}^{q_a(\cdot)} \underbrace{\frac{dV(\psi_{z,x,m})}{dm}}_{<0} F(m)dm\Big) \\
&- \frac{x}{z\theta(x)}\lambda\Big(\underbrace{\frac{\partial x_b(\cdot)}{\partial z}}_{\substack{=0 \text{ because } x \text{ is} \\ \text{in big party region}}} \underbrace{\frac{dV(\psi_{z,x,x_b(\cdot)})}{dx_b(\cdot)}\bar{F}(x_b(\cdot))}_{>0} - \underbrace{\frac{\partial q_b(\cdot)}{\partial z}}_{=0 \text{ (exogenous)}} \underbrace{\frac{dV(\psi_{z,x,q_b(\cdot)})}{dq_b(\cdot)}\bar{F}(q_b(\cdot))}_{>0}\Big) \\
&- \frac{x}{z\theta(x)}\lambda\Big(- \underbrace{\frac{\partial q_a(\cdot)}{\partial z}}_{=0 \text{ (exogenous)}} \underbrace{\frac{dV(\psi_{z,x,q_a(\cdot)})}{dq_a(\cdot)}}_{<0} F(q_a(\cdot)) + \underbrace{\frac{\partial x_a(\cdot)}{\partial z}}_{\substack{>0 \text{ because } x \text{ is} \\ \text{in big party region}}} \underbrace{\frac{dV(\psi_{z,x,x_a(\cdot)})}{dx_a(\cdot)}}_{<0} F(x_a(\cdot))\Big) \\
&- \frac{x}{z\theta(x)}\lambda\Big(\underbrace{\int_{q_b(\cdot)}^{x_b(\cdot)} \frac{d^2V(\psi_{z,x,m})}{dmdz}\bar{F}(m)dm}_{<0} - \underbrace{\int_{x_a(\cdot)}^{q_a(\cdot)} \frac{d^2V(\psi_{z,x,m})}{dmdz}F(m)\,dm}_{<0}\Big) \qquad \text{(E.4)}
\end{aligned}
$$

All of the terms in the first four lines in equation E.1 are non-negatively signed. The fifth line is negatively signed, but I show below that the sum of the terms in the second and fifth lines is greater than zero. It turns out that, among the politician types that consider a type-x party as a "big party," the rent share that convinces a politician to join the party is increasing in politician's resources. So, the low-quality politicians are easier to control.

Summing the second and the fifth lines, we get

$$
\begin{aligned}
&\frac{x}{z\theta(x)}\lambda\frac{1}{\rho+\delta}\Big(\int_{q_b(\cdot)}^{x_b(\cdot)} \left[\frac{d}{dm}\left(\frac{\theta(m)}{m}\right) - \frac{d}{dx}\left(\frac{\theta(m)}{m}\right) + \frac{\psi'(m)}{z}\right]\bar{F}(m)dm \\
&- \frac{x}{z\theta(x)}\lambda\frac{1}{\rho+\delta}\Big(\int_{x_a(\cdot)}^{q_a(\cdot)} \left[-\frac{d}{dm}\left(\frac{\theta(m)}{m}\right) + \frac{d}{dm}\left(\frac{\theta(m)}{m}\right) + \frac{\psi'(m)}{z}\right]F(m)dm \\
&= \frac{x}{z\theta(x)}\lambda\frac{1}{\rho+\delta}\Big(\int_{q_b(\cdot)}^{x_b(\cdot)} \frac{\psi'(m)}{z}\bar{F}(m)dm - \int_{x_a(\cdot)}^{q_a(\cdot)} \frac{\psi'(m)}{z}F(m)dm\Big) \qquad \text{(E.5)}
\end{aligned}
$$

Next, note that

$$\int_{q_b(\cdot)}^{x_b(\cdot)} \psi'(m)\bar{F}(m)dm = \bar{F}(m)\psi(x)|_{q_b(\cdot)}^{x_b(\cdot)} + \int_{q_b(\cdot)}^{x_b(\cdot)} \psi(m)f(m)dm$$
$$= \bar{F}(x_b(\cdot))\psi(x_b(\cdot)) - \bar{F}(q_b(\cdot))\psi(q_b(\cdot)) + \int_{q_b(\cdot)}^{x_b(\cdot)} \psi(m)f(m)dm \quad \text{(E.6)}$$

and

$$\int_{x_a(\cdot)}^{q_a(\cdot)} \psi'(m)F(m)dm = \psi(m)F(m)|_{x_a(\cdot)}^{q_a(\cdot)} - \int_{x_a(\cdot)}^{q_a(\cdot)} \psi(m)f(m)dm$$
$$= \psi(q_a(\cdot))F(q_a(\cdot)) - \psi(x_a(\cdot))F(x_a(\cdot)) - \int_{x_a(\cdot)}^{q_a(\cdot)} \psi(m)f(m)dm \quad \text{(E.7)}$$

Substituting equations E.6 and E.7 into equation E.5, we have

$$\begin{aligned}
&\frac{x}{z\theta(x)}\lambda\frac{1}{\rho+\delta}\Big(\int_{q_b(\cdot)}^{x_b(\cdot)} \left[\frac{d}{dm}\left(\frac{\theta(m)}{m}\right) - \frac{d}{dx}\left(\frac{\theta(m)}{m}\right) + \frac{\psi'(m)}{z}\right]\bar{F}(m)dm \\
&- \frac{x}{z\theta(x)}\lambda\frac{1}{\rho+\delta}\Big(\int_{x_a(\cdot)}^{q_a(\cdot)} \left[-\frac{d}{dm}\left(\frac{\theta(m)}{m}\right) + \frac{d}{dm}\left(\frac{\theta(m)}{m}\right) + \frac{\psi'(m)}{z}\right]F(m)dm \\
&= \frac{x}{z\theta(x)}\lambda\frac{1}{\rho+\delta}\Big(\int_{q_b(\cdot)}^{x_b(\cdot)} \frac{\psi'(m)}{z}\bar{F}(m)dm - \int_{x_a(\cdot)}^{q_a(\cdot)} \frac{\psi'(m)}{z}F(m)dm\Big) \\
&= \frac{x}{z^2\theta(x)}\lambda\frac{1}{\rho+\delta}\Big(\bar{F}(x_b(\cdot))\psi(x_b(\cdot)) - \bar{F}(q_b(\cdot))\psi(q_b(\cdot)) + \int_{q_b(\cdot)}^{x_b(\cdot)} \psi(m)f(m)dm \\
&- \psi(q_a(\cdot))F(q_a(\cdot)) + \psi(x_a(\cdot))F(x_a(\cdot)) + \int_{x_a(\cdot)}^{q_a(\cdot)} \psi(m)f(m)dm\Big) \\
&> 0 \qquad \text{(E.8)}
\end{aligned}$$

where the last inequality follows because $\int_{q_b(\cdot)}^{x_b(\cdot)} \psi(m)f(m)dm = E[\psi(m)|m \in (q_b(\cdot), x_b(\cdot))] > \psi(q_b(\cdot))$ and $F(q_b(\cdot))\psi(q_b(\cdot)) > \psi(q_a(\cdot))F(q_a(\cdot))$.

Case 2: A type-z politician considers a type-x party as a small party

If a politician considers a type-x party as a small party, the derivative of equation E.1

with respect to z is

$$\begin{aligned}
\frac{\partial\phi^l(\cdot)}{\partial z} &= \underbrace{-\frac{x}{z^2\theta(x)}[(1+\rho)\theta(z)-\psi(x)]}_{>0 \text{ by assumption 1}} + \underbrace{\frac{x}{z\theta(x)}(1+\rho)\theta'(z)}_{>0} \\
&+ \frac{x}{z^2\theta(x)}\lambda\Big(\int_{q_b(\cdot)}^{x_b(\cdot)} \underbrace{\frac{dV(\psi_{z,x,m})}{dm}}_{>0}\bar{F}(m)dm - \int_{x_a(\cdot)}^{q_a(\cdot)} \underbrace{\frac{dV(\psi_{z,x,m})}{dm}}_{<0}F(m)dm\Big) \\
&- \frac{x}{z\theta(x)}\lambda\Big(\underbrace{\frac{\partial x_b(\cdot)}{\partial z}}_{\substack{>0 \text{ because } x \text{ is} \\ \text{in small party region}}} \underbrace{\frac{dV(\psi_{z,x,x_b(\cdot)})}{dx_b(\cdot)}\bar{F}(x_b(\cdot))}_{>0} - \underbrace{\frac{\partial q_b(\cdot)}{\partial z}}_{=0 \text{ (exogenous)}} \underbrace{\frac{dV(\psi_{z,x,q_b(\cdot)})}{dq_b(\cdot)}\bar{F}(q_b(\cdot))}_{>0}\Big) \\
&- \frac{x}{z\theta(x)}\lambda\Big(-\underbrace{\frac{\partial q_a(\cdot)}{\partial z}}_{=0 \text{ (exogenous)}} \underbrace{\frac{dV(\psi_{z,x,q_a(\cdot)})}{dq_a(\cdot)}}_{<0}F(q_a(\cdot)) + \underbrace{\frac{\partial x_a(\cdot)}{\partial z}}_{\substack{=0 \text{ because } x \text{ is} \\ \text{in small party region}}} \underbrace{\frac{dV(\psi_{z,x,x_a(\cdot)})}{dx_a(\cdot)}}_{<0}F(x_a(\cdot))\Big) \\
&- \frac{x}{z\theta(x)}\lambda\Big(\underbrace{\int_{q_b(\cdot)}^{x_b(\cdot)} \frac{d^2V(\psi_{z,x,m})}{dmdz}\bar{F}(m)dm}_{<0} - \underbrace{\int_{x_a(\cdot)}^{q_a(\cdot)} \frac{d^2V(\psi_{z,x,m})}{dmdz}F(m)}_{<0}dm\Big) \qquad \text{(E.9)}
\end{aligned}$$

In this case, the terms on the first, second and fourth lines are non-negatively signed, whereas the terms on the third and the fifth lines are negatively signed. To rewrite the negative term on the third line, notice that

$$\begin{aligned}
&V(z,\phi^{l^*}(z,x_a(\cdot)),\phi^{l^*}(z,x_a(\cdot)),x_a(\cdot)) = V(z,\phi^{l^*}(z,x_b(\cdot)),\phi^{l^*}(z,x_b(\cdot)),x_b(\cdot)) \\
&\implies \frac{z\theta(x_a(\cdot))}{x_a(\cdot)} + \psi(x_a(\cdot)) = \frac{z\theta(x_b(\cdot))}{x_b(\cdot)} + \psi(x_b(\cdot) \\
&\implies \frac{\theta(x_a(\cdot))}{x_a(\cdot)} + \frac{d}{dx_a}\left(\frac{z\theta(x_a(\cdot))}{x_a(\cdot))} + \psi'(x_a(\cdot)\right)\frac{dx_a(\cdot)}{dz} \\
&= \frac{\theta(x_b(\cdot))}{x_b(\cdot)} + \frac{d}{dx_b}\left(\frac{z\theta(x_b(\cdot))}{x_b(\cdot))} + \psi'(x_b(\cdot)\right)\frac{dx_b(\cdot)}{dz} \qquad \text{(E.10)}
\end{aligned}$$

When the politician is in a "small" party, we have that $\frac{dx_a(\cdot)}{dz} = 0$ because the small-party switching threshold of a slightly more resourceful politician is the same as a type-z politician (they are willing to switch to any smaller party). Then, we have that

$$\frac{dx_b(\cdot)}{dz} = \frac{\frac{\theta(x_a(\cdot))}{x_a(\cdot)} - \frac{\theta(x_b(\cdot))}{x_b(\cdot)}}{\left(\frac{z\theta(x_b(\cdot))}{x_b(\cdot))} + \psi'(x_b(\cdot)\right)} > 0 \qquad \text{(E.11)}$$

So, the negative term on the third line can be written as

$$-\frac{x}{z\theta(x)}\lambda\frac{dx_b(\cdot)}{dz}\frac{dV(\psi_{z,x,x_b(\cdot)})}{dx_b(\cdot)}\bar{F}(x_b(\cdot)) = -\frac{x}{z\theta(x)}\frac{\lambda}{\rho+\delta}\left(\frac{\theta(x_a(\cdot))}{x_a(\cdot)} - \frac{\theta(x_b(\cdot))}{x_b(\cdot)}\right)\bar{F}(x_b(\cdot)) \tag{E.12}$$

The sum of the second and fifth lines were derived in equation E.8. Summing E.12 with E.8, we get

$$\begin{aligned}
&\psi(x_b(\cdot)) - F(x_b(\cdot))\psi(x_b(\cdot)) - \psi(q_b(\cdot)) + F(q_b(\cdot))\psi(q_b(\cdot))\\
&- \psi(q_a(\cdot))F(q_a(\cdot)) + \psi(x_a(\cdot))F(x_a(\cdot))\\
&+ \int_{q_b(\cdot)}^{x_b(\cdot)}\psi(m)f(m)dm + \int_{x_a(\cdot)}^{q_a(\cdot)}\psi(m)f(m)dm\\
&+ \left(-\frac{z\theta(x_a(\cdot))}{x_a(\cdot)} + \frac{z\theta(x_b(\cdot))}{x_b(\cdot)}\right)\bar{F}(x_b(\cdot))
\end{aligned} \tag{E.13}$$

and rearranging the terms yield

$$\begin{aligned}
&\left(-\frac{z\theta(x_a(\cdot))}{x_a(\cdot)} + \frac{z\theta(x_b(\cdot))}{x_b(\cdot)} + \psi(x_b(\cdot)) - \psi(x_a(\cdot))\right)\bar{F}(x_b(\cdot))\\
&+ \psi(x_a(\cdot))(\bar{F}(x_b(\cdot)) + F(x_a(\cdot)))\\
&- \bar{F}(q_b(\cdot))\psi(q_b(\cdot)) - \psi(q_a(\cdot))F(q_a(\cdot))\\
&+ \int_{q_b(\cdot)}^{x_b(\cdot)}\psi(m)f(m)dm + \int_{x_a(\cdot)}^{q_a(\cdot)}\psi(m)f(m)dm\\
=&\psi(x_a(\cdot))(\bar{F}(x_b(\cdot)) + F(x_a(\cdot))) + \psi(x)\\
&- \bar{F}(q_b(\cdot))\psi(q_b(\cdot)) - \psi(q_a(\cdot))F(q_a(\cdot))\\
&+ \int_{q_b(\cdot)}^{x_b(\cdot)}\psi(m)f(m)dm + \int_{x_a(\cdot)}^{q_a(\cdot)}\psi(m)f(m)dm\\
&> 0
\end{aligned} \tag{E.14}$$

where we use that $\frac{z\theta(x_a(\cdot))}{x_a(\cdot)} + \psi(x_a(\cdot)) = \frac{z\theta(x_b(\cdot))}{x_b(\cdot)} + \psi(x_b(\cdot))$. The last inequality follows because $\int_{q_b(\cdot)}^{x_b(\cdot)}\psi(m)f(m)dm = E[\psi(m)|m \in (q_b(\cdot), x_b(\cdot))] > \psi(q_b(\cdot))$ and $F(q_b(\cdot))\psi(q_b(\cdot)) > psi(q_a(\cdot))F(q_a(\cdot))$.

Therefore, we have shown that $\frac{\partial\phi^l(\cdot)}{\partial z} > 0$ when a type-z politician considers a type-x party as a small party.

E.2 Returns to a bigger party for a leader

This section shows that $\frac{\Pi(z,1,x)}{dx} > 0$ in an equilibrium with $\phi^{l^*}(z,x) = 1, \forall z, x$. A type-$z$ politician's profitability to a type-x leader is

$$\begin{aligned}
\Pi(z,1,x) &= \frac{z\theta(x)}{x}\Big\{ \int_{x_a(\cdot)}^{x_b(\cdot)} (1-\phi^l(z,x,x',1,1))\mu(z,x'|x,\Phi^{l^*}(z,x))dx' \\
&+ (1-\phi^l(z,x,0,1,0))\mu(z,0|x,\Phi^{l^*}(z,x))\Big\},
\end{aligned}$$

with derivative

$$\begin{aligned}
\frac{\Pi(z,1,x)}{dx} &= \frac{d}{dx}\Big(\frac{z\theta(x)}{x}\Big)\Big\{ \int_{x_a(\cdot)}^{x_b(\cdot)} (1-\phi^l(z,x,x',1,1))\mu(z,x'|x,\Phi^{l^*}(z,x))dx' \\
&+ (1-\phi^l(z,x,0,1,0))\mu(z,0|x,\Phi^{l^*}(z,x))\Big\}, \\
&+ \frac{z\theta(x)}{x}\Big\{ \frac{dx_b(\cdot)}{dx}\Big(\underbrace{(1-\phi^l(z,x,x_b(\cdot),1,1))}_{=0} \mu(z,x_b(\cdot)|x,\Phi^{l^*}(z,x)\Big) \\
&- \frac{dx_a(z,x,\phi^{l^*}(z,x))}{dx}\Big(\underbrace{(1-\phi^l(z,x,x_a(\cdot),1,1))}_{=0} \mu(z,x_a(\cdot)|x,\Phi^{l^*}(z,x)\Big) \\
&+ \int_{x_a(\cdot)}^{x_b(\cdot)} - \underbrace{\frac{d\phi^l(z,x,x',1,1)}{dx}}_{<0} \mu(z,x'|x,\Phi^{l^*}(z,x))dx' \\
&+ \int_{x_a(\cdot)}^{x_b(\cdot)} (1-\phi^l(z,x,x',1,1)) \underbrace{\frac{d\mu(z,x'|x,\Phi^{l^*}(z,x))}{dx}}_{=0} dx' \\
&- \underbrace{\frac{d\phi^l(z,x,0,\phi^{l^*}(z,x))}{dx}}_{<0} \mu(z,0|x,\Phi^{l^*}(z,x)) \\
&+ (1-\phi^l(z,x,0,\phi^{l^*}(z,x))) \underbrace{\frac{d\mu(z,0|x,\Phi^{l^*}(z,x))}{dx}}_{=0} \Big\}
\end{aligned}$$

where the third and fourth lines are zero because in a Nash equilibrium, when types $x_a(\cdot)$ and $x_b(\cdot)$ parties compete for the politician's services, they both pay him a rent share of 1, i.e., $\phi^l(z,x,x_a(\cdot),1,1) = \phi^l(z,x,x_b(\cdot),1,1) = 1$. Moreover, the sixth and the eighth lines are also zero because the joint densities of type-(z,x') and type-$(z,0)$ politicians in type-x

parties,

$$\mu_{z,x',x}(z,x'|x,\Phi^{l^*}(z,x)) = \frac{2(1+\kappa)\kappa}{[1+\kappa[\bar{F}(x_b(z,x',\phi^{l^*}(z,x'))) + F(x_a(z,x',\phi^{l^*}(z,x')))]]^3} f(x')\tilde{\ell}(z),$$

and

$$\mu_{z,0,x}(z,0|x,\Phi^{l^*}(z,x)) = \frac{1}{[1+\kappa[F(x_a(z)+\bar{F}(x_b(z))]]}\tilde{\ell}(z),$$

do not depend on x as long as a type-z politician derives a greater value from membership in a type-x party then membership in a type-x' party or being an independent. So, we have

$$\begin{aligned}
\frac{\Pi(z,1,x)}{dx} &= \int_{x_a(\cdot)}^{x_b(\cdot)} \frac{d}{dx}\Big(\frac{z\theta(x)}{x}\Big)\mu(z,x'|x,\Phi^{l^*}(z,x))dx' \\
&+ \Big\{\int_{x_a(\cdot)}^{x_b(\cdot)} \underbrace{-\Big(\frac{d}{dx}\Big(\frac{z\theta(x)}{x}\Big)\phi^l(z,x,x',\phi^{l^*}(z,x)) - \frac{z\theta(x)}{x}\frac{d\phi^l(z,x,x',1,1)}{dx}\Big\}}_{A} \\
&\times \mu(z,x'|x,\Phi^{l^*}(z,x))dx' \\
&+ \mu(z,0|x,\Phi^{l^*}(z,x))\underbrace{\Big\{-\frac{d}{dx}\Big(\frac{z\theta(x)}{x}\Big)\phi^l(z,x,0,1,0) - \frac{z\theta(x)}{x}\frac{d\phi^l(z,x,0,\phi^{l^*}(z,x))}{dx}\Big\}}_{B} \\
&+ \mu(z,0|x,\Phi^{l^*}(z,x))\frac{d}{dx}\Big(\frac{z\theta(x)}{x}\Big) \qquad \text{(E.15)}
\end{aligned}$$

Substituting for $\frac{d\phi^l(z,x,x',1,0)}{dx}$ using equation E.3, the expressions denoted by A and B can be rewritten as

$$\begin{aligned}
A = B &= -\phi^l(z,x,x',1,0)\underbrace{\Big(\frac{z\theta'(x)x - z\theta(x)}{x^2} + \frac{d}{dx}\Big(\frac{x}{z\theta(x)}\Big)\frac{z\theta(x)}{x}\frac{z\theta(x)}{x}\Big)}_{=0} \\
&- \frac{z\theta(x)}{x}\frac{x}{z\theta(x)}\Big\{-\underbrace{\psi'(x)}_{>0} - \lambda\underbrace{\frac{dx_b(\cdot)}{dx}}_{>0}\underbrace{\frac{dV(z,1,1,x_b(\cdot))}{dx_b(\cdot)}}_{>0}F(x) \\
&- \lambda\underbrace{\frac{dx_a(\cdot)}{dx}}_{<0}\underbrace{\frac{dV(z,1,1,x_a(\cdot))}{dx_a(\cdot)}}_{<0}F(x_a(\cdot))\Big\} \qquad ¿
\end{aligned}$$

0(E.16)

Using these values, the derivative becomes

$$
\begin{aligned}
\frac{\Pi(z,1,x)}{dx} &= \int_{x_a(\cdot)}^{x_b(\cdot)} \frac{d}{dx}\Big(\frac{z\theta(x)}{x}\Big)\mu(z,x'|x,\Phi^{l^*}(z,x))dx' \\
&+ \mu(z,0|x,\Phi^{l^*}(z,x))\frac{d}{dx}\Big(\frac{z\theta(x)}{x}\Big) \\
&+ \Big\{\int_{x_a(\cdot)}^{x_b(\cdot)}\Big\{\underbrace{\psi'(x)}_{>0} + \lambda\underbrace{\frac{dx_b(\cdot)}{dx}}_{>0}\underbrace{\frac{dV(z,1,1,x_b(\cdot))}{dx_b(\cdot)}}_{>0}F(x_b(\cdot)) \\
&+ \lambda\underbrace{\frac{dx_a(\cdot)}{dx}}_{<0}\underbrace{\frac{dV(z,1,1,x_a(\cdot))}{dx_a(\cdot)}}_{<0}F(x_a(\cdot))\Big\}\times\mu(z,x'|x,\Phi^{l^*}(z,x))dx' \\
&+ \mu(z,0|x,\Phi^{l^*}(z,x))\Big\{\underbrace{\psi'(x)}_{>0} + \lambda\underbrace{\frac{dx_b(\cdot)}{dx}}_{>0}\underbrace{\frac{dV(z,1,1,x_b(\cdot))}{dx_b(\cdot)}}_{>0}F(x_b(\cdot)) \\
&+ \lambda\underbrace{\frac{dx_a(\cdot)}{dx}}_{<0}\underbrace{\frac{dV(z,1,1,x_a(\cdot))}{dx_a(\cdot)}}_{<0}F(x_a(\cdot))\Big\}
\end{aligned} \tag{E.17}
$$

Only the first two terms in equation E.17 are negative, as $\frac{d}{dx}\Big(\frac{z\theta(x)}{x}\Big) < 0$. This equation can further be rearranged as

$$
\begin{aligned}
\frac{\Pi(z,1,x)}{dx} &= \Big\{\int_{x_a(\cdot)}^{x_b(\cdot)} \mu(z,x'|x,\Phi^{l^*}(z,x))\times\Big\{\underbrace{\frac{d}{dx}\Big(\frac{z\theta(x)}{x}\Big)}_{<0} + \underbrace{\psi'(x)}_{>0} \\
&+ \lambda\underbrace{\frac{dx_b(\cdot)}{dx}}_{>0}\underbrace{\frac{dV(z,1,1,x_b(\cdot))}{dx_b(\cdot)}}_{>0}F(x_b(\cdot)) + \lambda\underbrace{\frac{dx_a(\cdot)}{dx}}_{<0}\underbrace{\frac{dV(z,1,1,x_a(\cdot))}{dx_a(\cdot)}}_{<0}F(x_a(\cdot))\Big\}dx' \\
&+ \mu(z,0|x,\Phi^{l^*}(z,x))\Big\{\underbrace{\frac{d}{dx}\Big(\frac{z\theta(x)}{x}\Big)}_{<0} + \underbrace{\psi'(x)}_{>0} \\
&+ \lambda\underbrace{\frac{dx_b(\cdot)}{dx}}_{>0}\underbrace{\frac{dV(z,1,1,x_b(\cdot))}{dx_b(\cdot)}}_{>0}F(x_b(\cdot)) + \lambda\underbrace{\frac{dx_a(\cdot)}{dx}}_{<0}\underbrace{\frac{dV(z,1,1,x_a(\cdot))}{dx_a(\cdot)}}_{<0}F(x_a(\cdot))\Big\}
\end{aligned} \tag{E.18}
$$

For all politicians who consider a type-x party as big, since $\frac{d}{dx}\Big(\frac{z\theta(x)}{x}\Big) + \psi'(x) > 0$, we have that $\frac{\Pi(z,1,x)}{dx} > 0$.